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**MATHEMATICS**

**Specialist Units 3 & 4**

**Test 5 – Applications of Differentiation**

**Semester 2 2019**

# 

**Section Two – Calculator Assumed**

Time allowed for this section

Working time for this section: 35 minutes

Marks available: 33 marks

## Material required/recommended for this section

##### To be provided by the supervisor

This Question/Answer booklet

Formula sheet

##### To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

## Important note to candidates

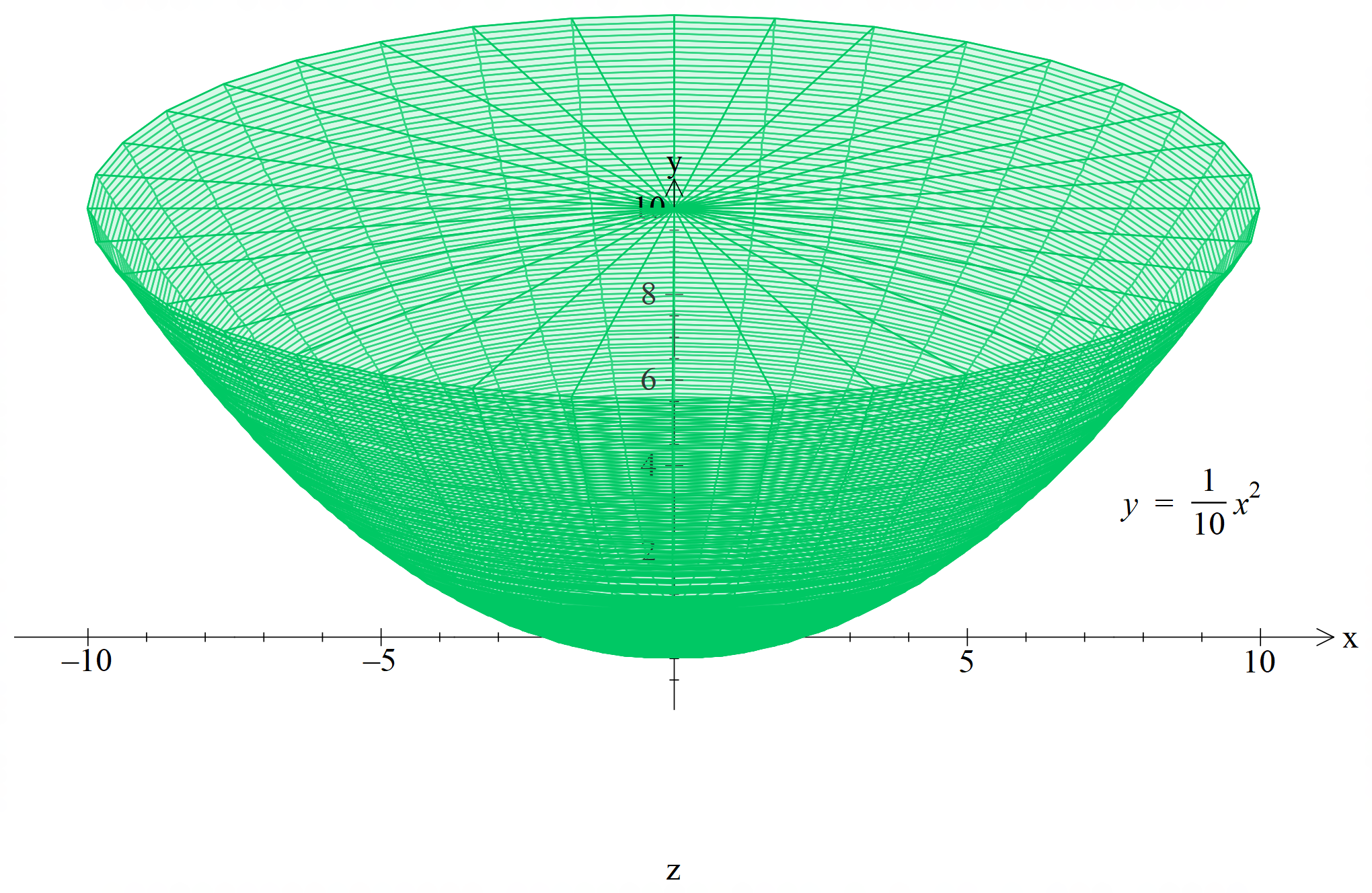
No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

1. (5 marks: 3, 2)  
   After seconds, the displacement centimetres of a small mass attached to a spring, oscillates about a fixed point according to the differential equation .

The initial velocity is 15 centimetres per second and the initial displacement is zero.

1. Determine the function that gives the displacement of the mass at time .
2. Calculate the distance the mass travels during the first 3 seconds.
3. (5 marks: 2, 3)

A dog’s drinking bowl is formed by revolving the curve , , through 360 about the y-axis. The units of and are in centimetres.



1. Show the expression connecting the height of water in the bowl and the volume of water in the bowl is given by .
2. The bowl is filled with water at a rate of 50mL per second. At what rate is the height of the water increasing at the time when the bowl contains 600 mL of water?
3. (7 marks)

Determine the values of the constants and if is a solution of the differential equation

1. (10 marks: 6, 4)

An ecology student is studying the repopulation of wild emus in Western Australia after a drought. She notices that the population growth rate is approximately logistic so that where is the population years after her study begins.

The carrying capacity is known to be 2550 emus, since this was the population before the drought.

Initially, the student finds that there are 310 emus. After 2 years she finds there are 390 emus.

1. Given that , use the separation of variables technique to determine the equation to find the population after years
2. Find the time at which the population growth rate is a maximum.
3. (6 marks: 3, 3)

The graph of is shown below:

A screenshot of a computer

Description automatically generated

1. By implicitly differentiating the given equation, obtain an equation relating and .

Note: Do not attempt to obtain as the subject of this equation.

1. Determine the coordinates of the point on the graph in the first quadrant ( where the gradient is horizontal.

**End of Test**

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You may use this space to extend or re-attempt an answer to a question or questions and should you do so then number the question(s) attempted and cross out any previous unwanted working.